AJOYIB TO‘G‘RI TO‘RTBURCHAKLAR

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Ushbu maqolada fazodagi ixtiyoriy nuqtadan, shu fazodagi ixtiyoriy to‘g‘ri to‘rtburchak qarama - qarshi uchlarigacha bo‘lgan masofalar kvadratlari yig‘indisi shu to‘rtburchakning qolgan ikki qarama-qarshi uchlarigacha bo‘lgan masofalari kvadratlari yig‘indisiga tengligi haqidagi teorema isbotlanilgan. Maqoladan geometriyada to‘g‘rito‘rtburchaklarga oid masalalarni echishda foydalanish mumkin.

Tayanch so‘zlar: nuqta, ikki nuqta orasidagi masofa, to‘g‘ri to‘rtburchak, to‘g‘ri to‘rtburchakning qarama-qarshi uchlari, nuqtadan to‘g‘ri to‘rtburchakning qarama-qarshi uchlarigach masofa, Pifagor teoremasi, nuqtadan proektsiya tekisligigach masofa.

Evklid geometriyasida masofalar o‘rtasidagi munosabatlar muhim ahamiyat kasb etadi.[1],[2] Ushbu maqolada ham to‘g‘ri to‘rtburchaklarning ajoyib xossalaridan biriga to‘xtalamiz.[3],[4] Unda tekislikdagi, fazodagi masofalar kvadratlari yig‘indisi haqida to‘xtalamiz.

Teorema. Fazoda ixtiyoriy nuqtadan ixtiyoriy to‘g‘ri to‘rtburchak qarama-qarshi uchlarigacha bo‘lgan masofalar kvadratlari yig‘indisi shu to‘g‘ri to‘rtburchakning qolgan ikki qarama-qarshi uchlarigacha bo‘lgan masofalari kvadratlari yig‘indisiga teng, ya’ni

$$a^2+c^2=b^2+d^2 \qquad (1)$$

formula orinli bo‘ladi.

Isbot. Ixtiyoriy M nuqta ABCD to‘g‘ri to‘rtburchak tekisligida yotmagan nuqta bo‘lsin. $MA=a, MB=b, MC=c, MD=d.$ M nuqtani to‘g‘ri to‘rtburchak tekisligiga proeksiyalaymiz, uning proeksiyasi N nuqta, $MN=h, h$ to‘g‘ri to‘rtburchak tekisligiga perpendikulyar. M nuqtani proeksiyasi to‘g‘ri to‘rtburchak ichiga tushsin. $NA=a_1, NB=b_1, NC=c_1, ND=d_1$ ular uchun teorema 1 o‘rinli bo‘ladi, shunga asosan $a_1^2+c_1^2=b_1^2+d_1^2$ munosabat o‘rinli bo‘ladi. Pifagor teoremasiga asosan

$$\begin{aligned} a^2 &= h^2+a_1^2 \\ b^2 &= h^2+b_1^2 \\ c^2 &= h^2+c_1^2 \\ d^2 &= h^2+d_1^2 \end{aligned} \qquad (2)$$

munosabatlar o‘rinli boladi. To‘g‘ri to‘rtburchak uchun quyidagi munosabat $a_1^2+c_1^2=b_1^2+d_1^2$ o‘rinli bo‘lganligidan, tenglikning har ikki tomoniga $2h^2$ ni qo‘shsak,

$$a_1^2+h^2+c_1^2+h^2=b_1^2+h^2+d_1^2+h^2$$

bunda (3) ni hisobga olib, $a^2+c^2=b^2+d^2$ ekanligi kelib chiqadi, ya’ni (2) formula o‘rinli bo‘ladi.

M nuqtaning proeksiyasi to‘g‘ri to‘rtburchak tekisligiga proektsiyalaganimizda uning proektsiyasi ABCD to‘g‘ri to‘rtburchakdan tashqarisiga tushgan holda ham yuqoridagi mulohazalar o‘rinli bo‘ladi.

Endi teorema 1 ni analitik isbotini keltiramiz. To‘g‘ri to‘rtburchakning tomonlarining tenglamasi perpendikulyar to‘g‘ri chiziqlar tenglamasidan iborat bo‘ladi:

1) $\begin{cases} y = kx + l_1, \\ y = (-1/k)x + m_1 \end{cases}$

ular kesishgan nuqta B ning koordinatalari bo'ladi

$x_2 = (k/(k^2+1))(m_1 - l_1), y_2 = (k^2/(k^2+1))(m_1 - l_1) + l_1, yoki (-1/(k^2+1))(m_1 - l_1) + m_1,$

ya'ni $B(x_2, y_2) = B((k/(k^2+1))(m_1 - l_1), (k^2/(k^2+1))(m_1 - l_1) + l_1))$

2) $\begin{cases} y = kx + l_2, l_2 < l_1 \\ y = (-1/k)x + m_1 \end{cases}$

ular kesishgan nuqta A ning koordinatalari bo'ladi

ya'ni $A(x_1, y_1) = A((k/(k^2+1))(m_1 - l_2), ((k^2/(k^2+1))(m_1 - l_2) + l_2))$

3) $\begin{cases} y = kx + l_1, \\ y = (-1/k)x + m_2, m_1 < m_2 \end{cases}$

ular kesishgan nuqta C ning koordinatalari bo'ladi

ya'ni $C(x_3, y_3) = C((k/(k^2+1))(m_2 - l_1), ((k^2/(k^2+1))(m_2 - l_1) + l_1))$

4) $\begin{cases} y = kx + l_2, \\ y = (-1/k)x + m_2, \end{cases}$

ular kesishgan nuqta D ning koordinatalari bo'ladi

ya'ni $D(x_4, y_4) = D((k/(k^2+1))(m_2 - l_2), ((k^2/(k^2+1))(m_2 - l_2) + l_2))$.

$M(x, y)$ ixtiyoriy nuqta bo'lsin, undan A nuqtagacha bo'lgan masofa [2]

$$MA = \sqrt{\left[\frac{k}{k^2+1}(m_1 - l_2) - x\right]^2 + \left[\frac{k^2}{k^2+1}(m_1 - l_2) + l_2 - y\right]^2} = a$$

$M(x, y)$ nuqtadan C nuqtagacha bo'lgan masofa

$$MC = \sqrt{\left[\frac{k}{k^2+1}(m_2 - l_1) - x\right]^2 + \left[\frac{k^2}{k^2+1}(m_2 - l_1) + l_1 - y\right]^2} = c$$

$M(x, y)$ nuqtadan B nuqtagacha bo'lgan masofa

$$MB = \sqrt{\left[\frac{k}{k^2+1}(m_1 - l_1) - x\right]^2 + \left[\frac{k^2}{k^2+1}(m_1 - l_1) + l_1 - y\right]^2} = b$$

$M(x, y)$ nuqtadan D nuqtagacha bo'lgan masofa

$$MD = \sqrt{\left[\frac{k}{k^2+1}(m_2 - l_1) - x\right]^2 + \left[\frac{k^2}{k^2+1}(m_2 - l_2) + l_2 - y\right]^2} = d$$

bo'ladi. Endi $a^2 + c^2 = b^2 + d^2$ muosabatni to'g'riligini analitik isbotini keltiramiz. Buning uchun a^2, c^2, b^2, d^2 larning ifodalarini yuqoridagi (2) tenglamaga qo'yib, *x,y* larga nisbatan ayniyat bo'lishini ko'rsatish etarli. Isbot qilinishi kerak bo'lgan tenglamani unga ekvivalent

$$a^2 - b^2 = d^2 - c^2. \qquad (3)$$

tenglikni isbotlashga keltiramiz.

U holda

$$a^2 - b^2 = \left[\frac{k}{k^2+1}(m_1 - l_2) - x\right]^2 + \left[\frac{k^2}{k^2+1}(m_1 - l_2) + l_2 - y\right]^2 -$$

$$-\left[\frac{k}{k^2+1}(m_1 - l_1) - x\right]^2 - \left[\frac{k^2}{k^2+1}(m_1 - l_1) + l_1 - y\right]^2$$

munosabatdan

$$a^2-b^2=\left\{\left[\frac{k}{k^2+1}(m_1-l_2)-x\right]^2-\left[\frac{k}{k^2+1}(m_1-l_1)-x\right]^2\right\}+$$
$$+\left\{\left[\frac{k^2}{k^2+1}(m_1-l_2)+l_2-y\right]^2-\left[\frac{k^2}{k^2+1}(m_1-l_1)+l_1-y\right]^2\right\}.$$

Bundan qisqa ko'paytirish

$$a^2-b^2=\left\{\left[(\frac{k}{k^2+1})(l_1-l_2)\right]\left[(\frac{k}{k^2+1})(2m_1-l_1-l_2)-2x\right]\right\}+$$
$$+\left\{\left[(\frac{k^2}{k^2+1})(l_1-l_2)-l_1+l_2\right]\left[(\frac{k^2}{k^2+1})(2m_1-l_1-l_2)+l_1+l_2-2y\right]\right\}=$$
$$=(\frac{k}{k^2+1})^2(l_1-l_2)(2m_1-l_1-l_2)-(\frac{k}{k^2+1})(l_1-l_2)2x+$$
$$+\left[(\frac{k^2+1-1}{k^2+1})(l_1-l_2)-(l_1-l_2)\right]\left[(\frac{k^2}{k^2+1})(2m_1-l_1-l_2)+(l_1+l_2-2y)\right]=$$
$$=(\frac{k}{k^2+1})^2(l_1-l_2)(2m_1-l_1-l_2)-(\frac{k}{k^2+1})(l_1-l_2)2x+$$
$$\left[(\frac{k^2+1-1}{k^2+1})(l_1-l_2)-(l_1-l_2)\right]\left[(\frac{k^2}{k^2+1})(2m_1-l_1-l_2)+(l_1+l_2-2y)\right]=$$
$$=(\frac{k}{k^2+1})^2(l_1-l_2)(2m_1-l_1-l_2)-(\frac{k}{k^2+1})(l_1-l_2)2x-$$
$$-\left[(\frac{1}{k^2+1})(l_1-l_2)-(l_1-l_2)\right]\left[(\frac{k^2}{k^2+1})(2m_1-l_1-l_2)+(l_1+l_2-2y)\right]=$$
$$=(\frac{k}{k^2+1})(l_1-l_2)2x-\left[(\frac{1}{k^2+1})(l_1+l_2-2y)\right]$$

formulasini qo'llab quyidagiga ega bo'lamiz:

$$a^2-b^2=\left\{\left[(\frac{k}{k^2+1})(l_1-l_2)\right]\left[(\frac{k}{k^2+1})(2m_1-l_1-l_2)-2x\right]\right\}+$$
$$+\left\{\left[(\frac{k^2}{k^2+1})(l_1-l_2)-l_1+l_2\right]\left[(\frac{k^2}{k^2+1})(2m_1-l_1-l_2)+l_1+l_2-2y\right]\right\}=$$
$$=(\frac{k}{k^2+1})^2(l_1-l_2)(2m_1-l_1-l_2)-(\frac{k}{k^2+1})(l_1-l_2)2x+$$
$$+\left[(\frac{k^2+1-1}{k^2+1})(l_1-l_2)-(l_1-l_2)\right]\left[(\frac{k^2}{k^2+1})(2m_1-l_1-l_2)+(l_1+l_2-2y)\right]=$$
$$=(\frac{k}{k^2+1})^2(l_1-l_2)(2m_1-l_1-l_2)-(\frac{k}{k^2+1})(l_1-l_2)2x+$$
$$\left[(\frac{k^2+1-1}{k^2+1})(l_1-l_2)-(l_1-l_2)\right]\left[(\frac{k^2}{k^2+1})(2m_1-l_1-l_2)+(l_1+l_2-2y)\right]=$$
$$=(\frac{k}{k^2+1})^2(l_1-l_2)(2m_1-l_1-l_2)-(\frac{k}{k^2+1})(l_1-l_2)2x-$$
$$-\left[(\frac{1}{k^2+1})(l_1-l_2)-(l_1-l_2)\right]\left[(\frac{k^2}{k^2+1})(2m_1-l_1-l_2)+(l_1+l_2-2y)\right]=$$
$$=(\frac{k}{k^2+1})(l_1-l_2)2x-\left[(\frac{1}{k^2+1})(l_1+l_2-2y)\right] \quad (4)$$

Endi d^2-c^2 ni yuqoridagidek hisoblaymiz:

$$d^2-c^2=\left[\frac{k}{k^2+1}(m_2-l_2)-x\right]^2+\left[\frac{k^2}{k^2+1}(m_2-l_2)+l_2-y\right]^2-$$
$$-\left[\frac{k}{k^2+1}(m_2-l_1)-x\right]^2-\left[\frac{k^2}{k^2+1}(m_2-l_1)+l_1-y\right]^2=$$
$$=\left\{\left[\frac{k}{k^2+1}(m_2-l_2)-x\right]^2-\left[\frac{k}{k^2+1}(m_2-l_1)-x\right]^2\right\}+$$
$$+\left\{\left[\frac{k^2}{k^2+1}(m_2-l_2)+l_2-y\right]^2-\left[\frac{k^2}{k^2+1}(m_2-l_1)+l_1-y\right]^2\right\}.$$

Bundan qisqa ko'paytirish formulasini qo'llab quyidagiga ega bo'lamiz:

$$d^2-c^2=\left\{\left[(\frac{k}{k^2+1})(l_1-l_2)\right]\left[(\frac{k}{k^2+1})(2m_2-l_1-l_2)-2x\right]\right\}+$$
$$+\left\{\left[(\frac{k^2}{k^2+1})(l_1-l_2)-l_1+l_2\right]\left[(\frac{k^2}{k^2+1})(2m_2-l_1-l_2)+l_1+l_2-2y\right]\right\}=$$
$$=(\frac{k}{k^2+1})^2(l_1-l_2)(2m_2-l_1-l_2)-(\frac{k}{k^2+1})(l_1-l_2)2x-$$
$$-\left[(\frac{1}{k^2+1})(l_1-l_2)\right]\left[(\frac{k^2}{k^2+1})(2m_2-l_1-l_2)+(l_1+l_2-2y)\right]=$$
$$=-(\frac{k}{k^2+1})(l_1-l_2)2x-\left[(\frac{1}{k^2+1})(l_1-l_2)(l_1+l_2-2y)\right] \quad (5)$$

(5) va (6) munosabatlardan (4) tenglik kelib chiqadi:

$$a^2-c^2=d^2-b^2=-(\frac{k}{k^2+1})(l_1-l_2)2x-\left[(\frac{1}{k^2+1})(l_1-l_2)(l_1+l_2-2y)\right].$$

Xuddi shunga o'xshash teorema-2 ni isbotini keltirish mumkin.

FOYDALANILGAN ADABIYOTLAR:

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