NATURAL SONLAR NATURAL DARAJALARI YIG‘INDILARINI TOPISHDA HOSILANING QOLLANILISHINING AHAMIYATI

Raxmonov J. T - Guliston davlat pedagogika instituti “Aniq fanlar” kafedrasi o‘qituvchisi.
Nurbayev Abdurashid Ravshanovich – GULDU o‘qituvchisi.
Xabibullayeva Muxlisa Xayrullo qizi - Guliston davlat pedagogika instituti Matematika yo‘nalishi 1-kurs talabasi

Annotatsiya: Biz bu maqolada, ya’ni ishimizda n ta natural sonlarni natural darajalari yig‘indisini topish masalasini tadqiq etishga bag‘ishladik. Bundan avval ko‘rilgan matematik induksiya, yig‘indini o‘ziga xos usulda topish va qo‘shimcha ravishda hosila yordamida yig‘indilarni topish masalasi ko‘rib chiqiladi. Ishimizning o‘ziga xosligi hosila yordamida topilgan yig‘indining ancha qulayligi ko‘rsatilgan.

Kalit so‘zlar: Sonli qatorlar, natural darajalar, matematik induksiya yordamida isbotlash, metod, formulalarni isbotlash, hosila yordamida isbotlash, summa orqali isbotlash, formulalarni keltirib chiqarishni yo‘llari, yig‘indining qiymati.

Maqolani mazmuni shundan iboratki, har qanday natural darajali sonlarni yig‘indisini topishni qisqa formulalarini keltirib chiqarishni bir nechta usul hamda metodlarini sizlarga isbotlab ko‘rsatamiz.

Bizga har xil sonli qatorlar ma’lum bo‘lsin deylik

$$S_1(n)=1+2+3+4+\ldots\ldots\ldots\ldots\ldots\ldots+n=\frac{n(n+1)}{2} \quad (1)$$

$$S_2(n)=1^2+2^2+3^2+\ldots\ldots\ldots\ldots\ldots\ldots+n^2+\frac{n(n+1)(2n+1)}{6} \quad (2)$$

Formulalar elementar matematikada mavjud (1) formula arifmetik progressiyani yig‘indisi deb ataladi, ammo uni qanday kelib chiqishini hamma o‘quvchilar hamda o‘qituvchilar ham biladi deya olmaymiz. Tepada biz to‘xtalmoqchi bo‘lgan, sonli qatorlarni ko‘pchilik natural sonlar va ularning natural darajalari yig‘indilari formulalari deb biladi. Endi biz sizlarga shu formulalarni qanday kelib chiqqanligini ko‘rsatib o‘tmoqchimiz.

Birinchi metod: “**Matematik induksiya**” [1-2]

$$\sum_{iq1}^{n} i^2 + 1^2 + 2^2 + 3^2 + \cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots + n^2$$

$$\sum_{kq1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$k+1$ deb yuqoridagi formulaga 1 ni qo‘yib tekshirib ko‘ramiz.

$$1^{2q\frac{1(1Q1)(2*1Q1)}{6}}q1$$

nqk da o‘rinli deymiz va $n=k+1$ da isbotlab ko‘ramiz.

$$1^2 + \cdots k^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2 =$$
$$q(k+1)\left[\frac{k(2k+1)}{6} + (k+1)\right] = (k+1)\left[\frac{2k^2+7k+6}{6}\right] =$$
$$= \left(\frac{(k+1)(k+2)(2k+3)}{6}\right)$$

Ikkinchi metod:

Yuqoridagi (1) formuladan foydalanib biz (2) formulamizni yig'indi yordamida keltirib chiqarishni ko'ramiz.

$$\sum_{kq1}^{n}((\boldsymbol{k+1})^{\boldsymbol{3}} - \boldsymbol{k^3}) = \boldsymbol{2^3 - 1^3 + 3^3 - 2^3 + 4^3 - 3^3 + \cdots}$$
$$\ldots. + - \mathbf{n^3} = \mathbf{(n+1)^3 - 1^3} \qquad (5)$$

Yuqoridagi (5) formulada 2^3-1^3+3^3-2^3+……+$(n+1)^3$-n^3 lar k ni o'rniga 1^3 dan $(n+1)^3$-n^3 gacha bo'lgan qiymatlardir.

Endi esa formulani isbotlashni davom ettiramiz.

$$\sum_{kq1}^{n}((k+1)^3 - k^3) +$$
$$\sum_{kq1}^{n}(k^3 + 3k^2 + 3k + 1 - k^3) = \sum_{kq1}^{n}(3k^2 + 3k + 1)$$

$$q3\sum_{kq1}^{n}k^2 + 3\sum_{kq1}^{n}k + \sum_{kq1}^{n}1\mathrm{q}\,3\sum_{kq1}^{n}k^2 + 3\frac{n(n+1)}{2} + n$$
$$= (n+1)^3 - 1^3,$$
$$3\sum_{kq1}^{n}k^2 q(n+1)^3 - \frac{3n(n+1)}{2} - (n+1)$$
$$3\sum_{kq1}^{n}k^2\, q(n+1)\left((n+1)^2 - \frac{3n}{2} - 1\right)$$
$$3\sum_{kq1}^{n}k^2 q(nQ1)[\frac{2n^2+4n+2-3n-2}{2}]$$
$$\sum_{kq1}^{n}\boldsymbol{k^2} = \frac{\boldsymbol{n(n+1)(2n+1)}}{\boldsymbol{6}}$$

Bu natural sonlarni kvadratlarini yig'indisini formulasini isboti edi.Endi esa 3 darajalarini yig'indisini ham keltirib chiqarishni ko'ramiz.

$$\sum_{kq1}^{n}((k+1)^4 - k^4) = -1^4 + 3^4 - 2^4 + \cdots + (n+1)^4 - n^4 = -n^4$$

$$\sum_{kq1}^{n}(4k^3+6k^2+4k+1)=4\sum_{kq1}^{n}k^3+6\sum_{kq1}^{n}k^2+4\sum_{kq1}^{n}k+\sum_{kq1}^{n}1$$

$$4\sum_{k=1}^{n}k^3+6\frac{n(n+1)(2n+1)}{6}+4\frac{n(n+1)}{2}+n=-n^4$$

$$4\sum_{kq1}^{n}k^3q(n+1)[(n+1)^3-n(2n+1)-2n-1]$$

$$\sum_{\boldsymbol{kq1}}^{\boldsymbol{n}}\boldsymbol{k^3q}(\frac{\boldsymbol{n(n+1)}}{\boldsymbol{2}})^{\boldsymbol{2}}$$

Mana uchinchi darajasini kelib chiqishini ham ko'rib bilib oldik. Endi esa keyingi usulni ko'ramiz.

Uchinchi metod: "Hosila yo'li bilan isbotlash"[3]

$$\sum_{\mathrm{i=1}}^{\mathrm{n}}\mathrm{i}^3=\mathrm{f(n)}$$

$$\left(\sum_{i=1}^{n}i^3\right)'=f'(n)+c$$

$$f'(n)=\sum_{i=1}^{n}3i^2+c'=3\sum_{iq1}^{n}i^2+c'$$

$$f'(n)=3\frac{n(n+1)(2n+1)}{6}+c'$$

$$f'(n)=\frac{2n^3+3n^2+n}{2}+c'$$

$$f(n)=\frac{n^4}{4}+\frac{n^3}{3}+\frac{n^2}{2}+c'n+c''$$

$$f(0)=c''=0$$

$$f(1)=1=\frac{1}{4}+\frac{1}{2}+\frac{1}{4}+c'$$

$$1=1+c' \qquad c'=0$$

$$\boldsymbol{f(n)=\frac{n^4}{4}+\frac{n^3}{3}+\frac{n^2}{2}=\frac{n^2}{2}(n^2+2n+1)=\frac{n^2(n+1)^2}{4}=\left(\frac{n(n+1)}{2}\right)^2} \qquad (6)$$

Endi 4 darajalisini ham ko'rib chiqamiz.

$$\sum_{i=1}^{n}i^4=f(n)$$

$$\left(\sum_{i=1}^{n}i^4\right)'=f(n)+c$$

$$f'(n)=\sum_{i=1}^{n}4i^3+c'+4\sum_{i=1}^{n}i^3+c'$$

$$f'(n)=4\frac{n^2(n+1)^2}{4}+c'$$

$$f'(n)=n^4+2n^3+n^2+c'$$

$$f(n)=\frac{n^5}{5}+\frac{n^4}{4}+\frac{n^3}{3}+c'n+c''$$

$$f(0)=c''=0$$

$$f(1)=1=\frac{1}{5}+\frac{1}{2}+\frac{1}{3}+c'$$

$$1=\frac{31}{30}+c' \qquad c'=-\frac{1}{30}$$

$$\mathbf{F(n)=\frac{n^5}{5}+\frac{n^4}{4}+\frac{n^3}{3}-\frac{1}{30}=\frac{6n^5+15n^4+10n^3-1}{30}} \qquad \mathbf{(7)}$$

Hosila yordamida yig'indini topish natijasi avvalgi natijalarni rad etmaydi va avvalgi usullarga nisbatan soddaligi bilan ajralib turishi (7) formulada yaqqol ko'rinib turibdi[5]. Demak, bu kabi masalalar shu usulda ishlash, matematika o'qitish metodikasi fanida qo'llash taklif etiladi.

Xulosa. Biz bu maqolada, ya'ni ishimizda n ta natural sonlarni natural darajalari yig'indisini topish masalasini tadqiq etdik. Bundan avval ko'rilgan matematik induksiya, yig'indini o'ziga xos usulda topish va qo'shimcha ravishda hosila yordamida yig'indilarni topish masalasi ko'rib chiqilgan edi. Ishimizning o'ziga xosligi hosila yordamida topilgan yig'indining ancha qulayligi ko'rsatilgan. Uchta usul taqqoslangan holda metodik jihatdan natijaga erishilgan deb hisoblaymiz.

FOYDALANILGAN ADABIYOTLAR:

1. T.Azlarov va boshqalar "Matematikadan qo'llanma 2" T. 1994-yil.
2. T.Azlarov, X.Mansurov "Matematika analiz 1-qism" 1-tom 1994-yil
3. http:g'g' codeforce.com
4. Xamedova "Matematika". Boshlang'ich ta'lim yo'nalishi talabalari uchun darslik.200.
5. G. Xudoyberganov va b. "Matematik analizdan ma'ruzalar" Toshkent 2010 y. Voris nashriyoti