BAZI KECHIKUVCHI ARGUMENTLI XUSUSIY HOSILALI KVAZICHIZIQLI TENGLAMA UCHUN ARALASH MASALA

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Maqolada kechikuvchi argumentli giperbolik turdagi kvazichiziqli quyidagi tenglamaning $Q = \{\tau \le t \le T, 0 \le x \le l_1, 0 \le y \le l_2, 0 \le z \le l_3\}$ sohada

$$\frac{\partial^2 u}{\partial t^2} = a^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + b^2\left(\frac{\partial^2 u(t-\tau,x,y,z)}{\partial x^2} + \frac{\partial^2 u(t-\tau,x,y,z)}{\partial y^2} + \frac{\partial^2 u(t-\tau,x,y,z)}{\partial z^2}\right) + f(t,x,y,z,u(t,x,y,z),u(t-\tau,x,y,z))$$

$$+f(t,x,y,z,u(t,x,y,z),u(t-\tau,x,y,z)) \qquad (1)$$

$(t,x,y,z)\epsilon E = \{0 \le t \le \tau, 0 \le x \le l_1, 0 \le y \le l_2, 0 \le z \le l_3\}$ bo'lganda boshlang'ich

$$\left.\begin{aligned} u(t,x,y,z) &= \varphi(t,x,y,z) \\ u_t(t,x,y,z) &= \varphi_t'(t,x,y,z) \end{aligned}\right\} \qquad (2)$$

berilgan boshlang'ich shartlarni va $\tau \le t \le T$ bo'lganda

$$\left.\begin{aligned} u(t,0,y,z) &= 0 \\ u(t,l_1,y,z) &= 0 \end{aligned}\right\} \; 0 \le y \le l_2, 0 \le z \le l_3, \qquad \left.\begin{aligned} u(t,x,0,z) &= 0 \\ u(t,x,l_2,z) &= 0 \end{aligned}\right\} \; 0 \le x \le l_1,$$

$$0 \le z \le l_3, \qquad \left.\begin{aligned} u(t,x,y,0) &= 0 \\ u(t,x,y,l_2) &= 0 \end{aligned}\right\} \; 0 \le x \le l_1 \quad 0 \le y \le l_2 \qquad (3)$$

bir jinsli chegaraviy shartlarni qanoatlantiruvchi yechimini chekli-ayirmali usulda topish masalasi o'rganilgan. Bu masalani klassik, deyarli va umumlashgan yechimlarini mavjudligi va yagonaligi masalasi [2] da muallif tomonidan tatqiq etilgan. Shuningdek koordinata ikkita bo'lganda (1)-(3) masalani sonli yechimlari [3] da oshkormas sxema asosida haydash usulida yechilgan. Q sohani to'rlaymiz: $t_n = n\tau$, $x_i = i\Delta_1$, $y_j = j\Delta_2$, $z_j = j\Delta_3$ deb, to'rlangan to'r funksiya uchun $u(t_n, x_i, y_j, z_k) = u_{ijk}^n$ belgilashlarni kiritamiz, n=0,1,2,…, M, i=0,1,2,…,N_1, j=0,1,2,…, N_2, k=0,1,2,…,

N_3, bunda $M\tau = T,\ N_1\Delta_1 = l_1,\ N_2\Delta_2 = l_2,\ N_3\Delta_3 = l_3$, $N_1,\ N_2,\ N_3$ natural sonlardir. (2) boshlang'ich shartlardan $(t,x,y,z)\epsilon E$ bo`lganda(n=0 va n=1 da)

$$u_{ijk}^0 = \varphi(0, x_i, y_j, z_k)\ , \quad \frac{u_{ijk}^1 - u_{ijk}^0}{\tau} \approx \varphi_t'(\tau, x_i, y_j, z_k) \qquad (4)$$

$$u_{ijk}^1 \approx u_{ijk}^0 + \tau\varphi_t'(\tau, x_i, y_j, z_k) = \varphi(0, x_i, y_j, z_k) + \tau\varphi_t'(\tau, x_i, y_j, z_k) \qquad (5)$$

(3) chegaraviy shartlardan

$$u_{0jk}^n = 0\ , \quad u_{Njk}^n = 0\ , \quad u_{i0k}^n = 0\ , \quad u_{iNk}^n = 0,\ u_{ij0}^n = 0\ , \quad u_{ijN}^n = 0 \qquad (6)$$

qiymatlarga ega bo'lamiz, bunda N mos ravishda $N_1,\ N_2,\ N_3$ ga teng. Demak Q sohaning yon tomonlari va asosida $u(t,x,y,z)$ ning qiymatlari berilgan. Yuqoridagilardan foydalanib Q sohaning ichki tugun nuqtalarida $u(t,x,y,z)$ ning sonli qiymatlarini topamiz.

Umumiylikka halal bermasdan quyidagi oshkormas sxemalardan foydalanishimiz mumkin:

$$\frac{u_{ijk}^{n+1} - 2u_{ijk}^n + u_{ijk}^{n-1}}{\tau^2} = a^2\left(\frac{u_{i+1,jk}^{n+1} - 2u_{ijk}^{n+1} + u_{i-1,jk}^{n+1}}{{\Delta_1}^2} + \frac{u_{i,j+1,k}^n - 2u_{ijk}^n + u_{i,j-1,k}^n}{{\Delta_2}^2} + \frac{u_{i,j,k+1}^n - 2u_{ijk}^n + u_{i,j,k-1}^n}{{\Delta_3}^2}\right) +$$

$$+b^2\left(\frac{u_{i+1,jk}^n - 2u_{ijk}^n + u_{i-1,jk}^n}{{\Delta_1}^2} + \frac{u_{i,j+1,k}^n - 2u_{ijk}^n + u_{i,j-1,k}^n}{{\Delta_2}^2} + \frac{u_{i,j,k+1}^n - 2u_{ijk}^n + u_{i,j,k-1}^n}{{\Delta_3}^2}\right) + f_{ijk}^n \qquad (7)$$

yoki

$$\frac{u_{ijk}^{n+1} - 2u_{ijk}^n + u_{ijk}^{n-1}}{\tau^2} = a^2\left(\frac{u_{i+1,jk}^n - 2u_{ijk}^n + u_{i-1,jk}^n}{{\Delta_1}^2} + \frac{u_{i,j+1,k}^{n+1} - 2u_{ijk}^{n+1} + u_{i,j-1,k}^{n+1}}{{\Delta_2}^2} + \frac{u_{i,j+1,k}^n - 2u_{ijk}^n + u_{i,j-1,k}^n}{{\Delta_3}^2}\right) +$$

$$+b^2\left(\frac{u_{i+1,jk}^n - 2u_{ijk}^n + u_{i-1,jk}^n}{{\Delta_1}^2} + \frac{u_{i,j+1,k}^n - 2u_{ijk}^n + u_{i,j-1,k}^n}{{\Delta_2}^2} + \frac{u_{i,j,k+1}^n - 2u_{ijk}^n + u_{i,j,k-1}^n}{{\Delta_3}^2}\right) + f_{ijk}^n \qquad (7_1)$$

yoxud

$$\frac{u_{ijk}^{n+1}-2u_{ijk}^{n}+u_{ijk}^{n-1}}{\tau^2}=a^2\left(\frac{u_{i+1,jk}^{n}-2u_{ijk}^{n}+u_{i-1,jk}^{n}}{\Delta_1{}^2}+\frac{u_{i,j+1,k}^{n}-2u_{ijk}^{n}+u_{i,j-1,k}^{n}}{\Delta_2{}^2}\right.$$

$$\left.+\frac{u_{i,j+1,k}^{n+1}-2u_{ijk}^{n+1}+u_{i,j-1,k}^{n+1}}{\Delta_3{}^2}\right)+$$

$$+b^2\left(\frac{u_{i+1,jk}^{n}-2u_{ijk}^{n}+u_{i-1,jk}^{n}}{\Delta_1{}^2}+\frac{u_{i,j+1,k}^{n}-2u_{ijk}^{n}+u_{i,j-1,k}^{n}}{\Delta_2{}^2}+\frac{u_{i,j,k+1}^{n}-2u_{ijk}^{n}+u_{i,j,k-1}^{n}}{\Delta_3{}^2}\right)+f_{ijk}^{n} \qquad (7_2)$$

oshkormas sxemalardan foydalanish maqsadga muvofiq. Bu yerda

$$f_{ijk}^{n}=f(n\tau,i\Delta_1,j\Delta_2,k\Delta_3,u(n\tau,i\Delta_1,j\Delta_2,k\Delta_3),u\big((k-1)\tau,i\Delta_1,j\Delta_2,k\Delta_3\big))$$

$\tau\leq t\leq 2\tau$ bo'lganda, yuqoridagi oshkormas sxemaga haydash usulini qo'llaymiz.

(7) sxemada n=1 bo'lganda haydash koeffitsientlarni to'g'ri usulda o'sish tartibida topib boriladi. Shu zaylda davom etib, u_{ijk}^{2} ni teskari usulda kamayish tartibida topib boriladi. Shunday qilib ikkinchi qatlamda u_{ijk}^{2} lar to'rning tugun nuqtalaridagi qiymatlar topiladi.

$2\tau\leq t\leq 3\tau$ bo'lganda, yuqoridagi (7) oshkormas sxemaga haydash usulini qo`llaymiz. n=2 bo'lganda barcha yuqoridagi jarayonlar takrorlanib uchinchi qatlamda u_{ijk}^{3} topiladi va h.k. u_{ijk}^{n} hisoblab chiqiladi.

Yuqoridagi (7_1) va (7_2) _ ayirmali sxemadan foydalanib ham u_{ijk}^{n} hisoblab chiqish mumkin.

Foydalanilgan adabiyotlar

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