TERMINAL TO‘PLAM QO‘ZG‘ALUVCHAN BO‘LGAN DINAMIK TIZIM MODELI UCHUN NISBIY BOSHQARILUVCHANLIK XOSSASI

Rahimov Boyhuroz Shermuhammadovich - Jizzax politexnika instituti katta o‘qituvchi
E-mail: raximovboyxoroz@gamil.com

Dinamik tizimlar uchun differentsial mansubliklar trayektoriyalari ning berilhgan terminal to‘plamga boshqariluvchanligi xossasi dolzarb muammolardan biridir. Bu muammo terminal to‘plam qo‘zg‘aluvchan (vaqtdan bog‘liq) bo‘lgan holda al’ohida ahamiyat kasb etadi.

Dinamik tizim modeli sifatida differentsial mansubliklarni qaraymiz. Differentsial mansubliklar deb

$$\frac{dx}{dt} \in F(t,x) \qquad (1)$$

ko‘rinishdagi munosobatlarga aytiladi, bu yerda x – tizim holatini ifodalovchi n-vektor, t - vaqt, $F(t,x)$ – ko‘p qiymatli akslantirish.

Ta’rif 1. Berilgan (1) differentsial mansublikning $x_1 \in R^n$ terminal holatga nisbatan boshqariluvchanlik to‘plami deb shunday $x_0 \in R^n$ nuqtalar to‘plamiga aytiladiki, ular uchun $x(t) = x(t,x_0)$, $T = [t_0,t_1]$, trayektoriyalar mavjud va $x(t_0) = x_0$, $x(t_1) = x_1$ shartlar bajariladi.

Ta’rif 2. Berilgan (1) differentsial mansublikning $M = M(t)$ terminal to‘plamga nisbatan boshqariluvchanlik to‘plami(yoki qisqacha, M -boshqariluvchanlik to‘plami) deb shunday $x_0 \in R^n$, $x_0 \notin M(t_0)$ nuqtalarga aytiladiki, ular uchun $x(t) = x(t,x_0)$, $T = [t_0,t_1]$ trayektoriya mavjud va $x(t_0) = x_0$, $x(t_1) \in M(t_1)$.

$W(M,F)$ – (1) differentsial mansublikning M - boshqariluvchanlik to‘plami; $X(t_0,t_1,x_0,F)$–(1) differentsial mansublikning $x_0 \in R^n$ nuqtadan chiquvchi trayektoriyalarining $t_1 > t_0$ vaqtda erishiladigan nuqtalari to‘plami.

(1) differentsial mansublikning M - boshqariluvchanlik to‘plami strukturaviy xossalarini quyidagi to‘plam xossalari orqali o‘rganamiz:

$$K(t_0,t_1,M,F) = \left\{\xi \in R^n : X_T(t_0,t_1,\xi,F) \cap M(t_1) \neq \varnothing\right\}, t_1 > t_0.$$

Agar $A: R^1 \to R^{nxn}$, $B: R^1 \to \Omega(R^n)$, aksalantirishlar mavjud bo‘lib, $A(t)x + B(t) \subset F(t,x)\ \forall (t,x) \in R^1 \times R^n$, bo‘lsa, (1) differentsial mansublikning M - boshqariluvchanlik to‘plami xossalari

$$\dot{x} \in A(t)x + B(t) \quad . \tag{2}$$

Quyidagi shartlar bajarilgan deb hisoblaymiz:

$A(t)$ matritsalar elementlari har bir $T=[t_0,t_1]\subset[t_0,+\infty]$ oraliqda o'lchovli funktiyalar va $\| A(t)\| \le a(t)$, bu yerda $a(\cdot)\in L_1(T)$;

$t \to B(t)\in \Omega(R^n)$ ko'p qiymatli akslantirish har bir $T=[t_0,t_1]\subset[t_0,+\infty]$ segmentda o'lchovli va $\| B(t)\| \le b(t)$, bu yerda $b(\cdot)\in L_1(T)$.

(2) differentsial mansublik erishiladigan to'plami $X(t_0,t_1,\xi,A,B)$ uchun

$$X(t_0,t_1,\xi,A,B)=\Phi(t,t_0)\xi+\int_{t_0}^{t_1}\Phi(t_1,\tau)B(\tau)d\tau\,,$$

formula o'rinlidir, bu yerda $\Phi(t,\tau)$ $\dot{x}\in A(t)x$ tenglamaning fundamental yechimlar matritsasi. $X(t_0,t_1,\xi,A,B)$ – R^n fazoning qavariq va kompakt to'plamidan iborat.

Teorema 1. $K(t_0,t_1,M,A,B)$ to'plam uchun quyidagi formula o'rinli:

$$K(t_0,t_1,M,A,B)=-\int_{t_0}^{t_1}\Phi(t_0,t)B(t)dt+\Phi(t_0,t_1)M(t_1). \tag{3}$$

Natija 1. Agar $M(t_1)$ qavariq va kompakt to'plam bo'lsat, u vaqda $K(t_0,t_1,M,A,B)$ to'plam ham qavariq va compact to'plam bo'ladi. Agar $M(t_1)$ va $convB(t)$, $t\in T=[t_0,t_1]$– qat'iy qavariq bo'lsa, $K(t_0,t_1,M,A,B)$ ham qat'iy qavariq bo'ladi.

Natija 2. Quyidagi formula o'rinli:

$$K(t_0,t_1,M,A,B)=K_0(t_0,t_1,A,B)+\Phi(t_0,t_1)M(t_1),$$

bu yerda $K_0(t_1,A,B)=K(t_0,t_1,\{0\},A,B)$.

$t_0<\bar{t}<t_1, \overline{M}(t)=K(t,t_1,M,A,B)$ bo'lsin. U holda:

$$K(t_0,t_1,M,A,B)=K(t_0,\bar{t},\overline{M},A,B)\ . \tag{4}$$

Shu oxitgi formuladan foydalanib, ko'rsatish mumkinki, agar $t_0<\bar{t}<t_1$ bo'lsa, ushbu munosabar

$$K(t_0,t_1,M,A,B)\subset K(t_0,\bar{t},M,A,B) \tag{5}$$

Faqat va faqat $K(\bar{t},t_1,M,A,B)\subset M(\bar{t})$ bo'lgandagina bajariladi.

Teorema 2. Agar $A(t)\equiv A$, $B(t)\equiv B$, $t\in T=[t_0,t_1]$ bo'lsa, u holda:

$$K(t_0,t_1,M,A,B)=\bigcup_{\xi\in M(t_1)}X(t_0,t_1,\xi,-A,-B). \tag{6}$$

Natija 3. $B(t)\equiv B$, $M(t)\equiv M$, $0\in \operatorname{int} B$, $0\in \operatorname{int} M$ bo'lsin. U vaqtda $W(M,A,B)$ to'plam $\xi=0$ nuqtaning qandaydir atrofini o'zida saqlaydi.

Ushbu ishda [1–4] ishlar tatqqot natijalari rivojlantirilgan. Ishda keltirilgan (3) formula asosida $K(t_0,t_1,M,A,B)$ to‘plam xossalari o‘rganilgan. (4) va (5) formulalar $K(t_0,t_1,M,A,B)$ to‘plam dinamikasi haqida tasavvur beradi. Teorema 2 da keltirilgan (6) formula statsionar (2) differentsial mansublik M-boshqariluvchanlik to‘plami va erishiladigan to‘plam orasidagi munosabatni ifodalaydi.

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