KOSHI TENGSIZLIGINING ALGEBRADA UCHRAYDIGAN EKSTREMAL TENGSIZLIKLARGA TATBIQI

Musurmanova Malohat Arslonbek qizi – Guliston davlat pedagogika instituti o‘qituvchi
e-mail: malohat3595@gmail.com
Anvarova Ismigul Ma’murjon qizi, Bo‘ronboyev Kattabek Ikrom o‘g‘li – Guliston davlat pedagogika instituti 2-bosqich talabalari

Yoshlarni chuqur bilimli, teran fikrlaydigan, har bir masalaga ijobiy yondashadigan insonlar bo`lib yetishishlarida fan olimpiadalari muhim ahamiyatga ega. Shu sababli ushbu maqola yoshlarning matematika fanidagi ekstremal masalalarga qiziqishlarini uyg‘otish, tengsizliklar yordamida yechiladigan ekstremal masalalar matematikaning eng jozibador masalalar to‘plami ekanligini o‘quvchi yoshlarga yetkazish uchun yozildi.

1-misol. $y = x^2 + x^6 + \frac{2}{x} + \frac{3}{x^2}, (x > 0)$ funksiyaning eng kichik qiymatini toping.

Isboti. Buning uchun berilgan funksiyani ushbu

$$y = x^2 + x^6 + \frac{1}{x} + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^2} + \frac{1}{x^2}$$

ko`rinishida yozib olamiz va Koshi tengsizligini qo‘llaymiz:

$$y = 7\sqrt[7]{x^2 \cdot x^6 \cdot \frac{1}{x} \cdot \frac{1}{x} \cdot \frac{1}{x^2} \cdot \frac{1}{x^2} \cdot \frac{1}{x^2}} = 7 .$$

Bu yerda tenglik $x = 1$ bo‘lganda bajariladi. Demak, berilgan funksiyaning eng kichik qiymati 7 ekan.

2-misol. Sotuvchi torozisi yelkalarining uzunliklari turli xil edi, ya’ni torozi buzilgan edi. Oluvchiga u $1\ kg$ shakarni 1-pallasiga qo‘yib, yana $1\ kg$ ni 2-pallaga qo‘yib o‘lchab jami $2\ kg$ shakar sotdi, bu holda kim yutqazadi?

Isboti. Bu misolni yechish uchun o`lchashdagi shakar og‘irliklarini m_1 va m_2 orqali belgilaymiz. «mexanikaning oltin qoidasi» ga ko‘ra

$1 \cdot d_1 = m_1 d_2$ va $m_2 d_1 = 1 \cdot d_2$

bo‘ladi. Bulardan

$$m_1 = \frac{d_1}{d_2} \text{ va } m_2 = \frac{d_2}{d_1}$$

$$m_1 + m_2 = \frac{d_1}{d_2} + \frac{d_2}{d_1} > 2\sqrt{\frac{d_1}{d_2}\frac{d_2}{d_1}} = 2$$

Kelib chiqadi, chunki $d_1 \neq d_2$. Demak, sotuvchi oluvchiga 2 kg dan ko'p shakar sotgan.

3-misol. Agar $a > 0, b > 0, c > 0, d > 0$ bo'lsa, ushbu

$$\frac{a}{b+c}+\frac{b}{c+d}+\frac{c}{d+a}+\frac{d}{a+b} \geq 2$$

tengsizlikni isbotlang

Isboti. Buning uchun berilgan tengsizlikning chap tomonini guruhlab yozamiz va Koshi tengsizligidan foydalanamiz:

$$\left(\frac{a}{b+c}+\frac{c}{d+a}\right)+\left(\frac{b}{c+d}+\frac{d}{a+b}\right)$$

$$=\frac{a(d+a)+c(b+c)}{(b+c)(d+a)}+\frac{b(a+b)+d(c+d)}{(c+d)(a+b)} \geq$$

$$\geq \frac{4[ad+a^2+bc+c^2]}{(a+b+c+d)^2}+\frac{4[ab+b^2+cd+d^2]}{(a+b+c+d)^2}=$$

$$=\frac{4[a^2+b^2+c^2+d^2+ab+bc+cd+ad]}{(a+b+c+d)^2}=$$

$$=\frac{2[(a+b+c+d)^2+(a-c)^2+(b-d)^2]}{(a+b+c+d)^2} \geq 2$$

4-misol. Agar $a, b, c > 0$ bo'lsa,

$$18a^2+12b^3+6c^6 \geq 36abc$$

ni isbotlang.

Isboti. $18a^2+12b^3+6c^6 \geq 36abc$ tengsizlikni ikkala tomonini 36 ga bo'lamiz.$\frac{a^2}{2}+\frac{b^3}{3}+\frac{c^6}{6} \geq abc$; $\frac{1}{r_1}+\frac{1}{r_2}+\cdots+\frac{1}{r_n}=1$ bo'lsa, Yung tengsizligi o'rinli.

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=1 \Rightarrow \frac{a^2}{2}+\frac{b^3}{3}+\frac{c^6}{6} \geq abc$$

tengsizlik o'rinli.

5-misol. $x, y, z > 0$ sonlar uchun $x^2+y^2+z^2 \geq xy+yz+zx$

tengsizlikni isbotlang.

Isboti. Koshi tengsizligining xususiy holi bo'lgan $x^2+y^2 \geq 2xy$, $y^2+z^2 \geq 2yz$, $x^2+z^2 \geq 2xz$ tengsizliklarni qo'shib, ushbu

$$(x^2+y^2)+(x^2+z^2)+(y^2+z^2)=2(x^2+y^2+z^2) \geq 2(xy+yz+zx)$$

tenglikni olamiz.

6-misol. Agar a, b, c musbat sonlar va $n, k \in N$ bo'lsa, u holda quyidagi

$$\frac{a^{n+k}}{b^k} + \frac{b^{n+k}}{c^k} + \frac{c^{n+k}}{a^k} \geq a^n + b^n + c^n$$

tengsizlikni isbotlang.

Isboti. Koshi tengsizligiga ko'ra,

$$\underbrace{\frac{a^{n+k}}{b^k} + \cdots + \frac{a^{n+k}}{b^k}}_{n} + \underbrace{b^n + b^n + \cdots b^n}_{k} \geq (n+k)\sqrt[n+k]{\frac{(a^{n+k})^n b^{nk}}{b^{nk}}} = (n+k)a^n$$

yoki

$$\mathrm{n} \cdot \frac{a^{n+k}}{b^k} + k \cdot b^n \geq (n+k)a^n$$

Xuddi shunday

$$\mathrm{n} \cdot \frac{b^{n+k}}{c^k} + k \cdot c^n \geq (n+k)b^n$$

$$\mathrm{n} \cdot \frac{c^{n+k}}{a^k} + k \cdot a^n \geq (n+k)c^n$$

tengsizliklarni hosil qilamiz. Bu tengsizliklarni hadma-had qo'shib,

$$\frac{a^{n+k}}{b^k} + \frac{b^{n+k}}{c^k} + \frac{c^{n+k}}{a^k} \geq a^n + b^n + c^n$$

ni hosil qilamiz.

7-misol. Musbat a, b, c sonlar $a^4 + b^4 + c^4 = 3$ tenglikni qanoatlantirsa,

$$\frac{1}{4-ab} + \frac{1}{4-ac} + \frac{1}{4-bc} \leq 1$$

tengsizlikni isbotlang.

Isboti. Koshi tengsizligidan foydalanib,

$$\frac{2}{4-ab} = \frac{4}{8-2ab} \leq \frac{4}{8-a^2-b^2} \leq \frac{1}{4-a^2} + \frac{1}{4-b^2}$$

tenglikni hosil qilamiz. Xuddi shunday

$$\frac{2}{4-ac} = \frac{4}{8-2ac} \leq \frac{4}{8-a^2-c^2} \leq \frac{1}{4-a^2} + \frac{1}{4-c^2}$$

$$\frac{2}{4-bc} = \frac{4}{8-2bc} \leq \frac{4}{8-b^2-c^2} \leq \frac{1}{4-b^2} + \frac{1}{4-c^2}$$

tengsizliklarni hosil qilamiz. Bu tengsizliklarni hama-had qo'shib,

$$\frac{1}{4-ab}+\frac{1}{4-ac}+\frac{1}{4-bc}\le\frac{1}{4-a^2}+\frac{1}{4-b^2}+\frac{1}{4-c^2}$$

tenglikni hosil qilamiz. Berilgan $a^4+b^4+c^4=3$ shartdan $a^2<2$ ekanligi kelib chiqadi. Bundan quyidagi

$$(a^2-1)^2(2-a^2)\ge 0 \Leftrightarrow \frac{1}{4-a^2}\le\frac{a^4+5}{18}$$

tengsizlik o‘rinli. Xuddi shunday

$$\frac{1}{4-b^2}\le\frac{b^4+5}{18}\ ;\ \frac{1}{4-c^2}\le\frac{c^4+5}{18}$$

tengsizliklar o‘rinli. Bu tengsizliklarni hadma-had qo‘shib,

$$\frac{1}{4-ab}+\frac{1}{4-ac}+\frac{1}{4-bc}\le\frac{1}{4-a^2}+\frac{1}{4-b^2}+\frac{1}{4-c^2}\le\frac{a^4+5}{18}+\frac{b^4+5}{18}+\frac{c^4+5}{18}=1$$

ekanligini hosil qilamiz.

Foydalanilgan adabiyotlar

1. H.Norjigitov, J.A.Bahramov “Matematik olimpiada masalalarini yechish uchun qo‘llanma” Toshkent, 2014-yil.147-bet.

2. H.Norjigitov, A.X.Nuraliyev “Matematikadan olimpiada masalalari” Toshkent, 2020-yil. 243-bet.

3. B.I.Abdullayev, J.U.Xujamova, R.A.Sharipov “Matematikadan olimpiada masalalari” Urganch, 2016-yil 60-bet.

C++ Code::Block KOMPILYATORIDA ODDIY DIFFERENSIAL